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Application of the polynomial chaos expansion method for forecasting structural response of two full-scale case studies

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ABSTRACT

Predicting the behavior of engineering structures with high accuracy remains a challenging task as a result of their continuous interaction with the immediate environment and varying operating conditions. In that context, forecasting tools are primarily focused on the creation of a model of a so-called baseline system. This established model serves as a foundation for identifying changes when new outputs deviate from the predictions made by the model. Physics-based numerical models, like the finite element method, often carry significant uncertainty stemming from assumptions regarding structural characteristics, environmental influences, and various loads affecting the system under study. Consequently, identifying the source of any existing discrepancies between obtained model results and measured data is difficult. This paper demonstrates a straightforward implementation of the polynomial chaos expansion method for the formulation of prognostic data-driven models targeted at tracking changes in continuously measured structural response. The method's effectiveness and positive features are showcased via practical application onto two full-scale engineering structures: a concrete arch dam and an industrial steel chimney. The models utilize environmental as well as response data collected over two years and two months of monitoring of these structures, respectively. The obtained results reveal the models' considerable potential as a long-term monitoring tool for autonomous assessment of structural behavior.

1 Introduction

As a four-dimensional concept, Structural Health Monitoring (SHM) enables real-time as well as spatial assessment of monitored systems [1-3]. It targets diagnosis of the current condition of structures, however also, based on recorded full data history, learning about load and response mechanisms, prognosis of evolution of damages, estimation of fatigue and residual life of structures [4]. SHM frameworks are commonly based on approaches which utilize either physics-based, data-driven or hybrid models [5]. In [6] the SHM paradigm is concisely described as continuous system identification of a physical or parametric model of the structure using time-dependent data.

The term “model” can be best summarized as a collection of numerical or analytical processes employed to mimic the behavior and response of a real-world system to various changing factors [7]. However, all mathematical models inherently include uncertainties [8]. These are related to: i) modeling errors caused by oversimplified assumptions for the modeled process, ii) numerical errors due to insufficient resolution of applied numerical methods, and iii) data errors linked to limited knowledge and availability of input data, the inherent variability of the system being studied [9]. Various uncertainty quantification tools deal with systems affected by stochastic variations in system parameters (data errors) by taking into account the evolution of the probability distribution of random inputs [10]. In contrast to traditional collocation methods used for uncertainty quantification, spectral methods are based on a fundamentally different concept. Rather than conducting multiple simulations on an established mathematical model, non-sampling approaches aim to construct a functional relationship between a model's output quantity and a random input [11]. While these Fourier-like series representations impose certain requirements on both the output and input parameters, they offer a much lower computational cost compared to the widely used Monte Carlo method and other sampling techniques, which encompass a more "local" nature and asymptotic convergence rates [11-12].

This paper is focused on one representative of the spectral methods class, the Polynomial Chaos Expansion (PCE). The PCE method enables the generation of data-driven models as an approximation of input–output relationship by casting the model response onto orthogonal polynomials, with relatively simple mathematical formulation

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and efficiently spent computer time [13]. Additionally, the PCE method provides a suitable mathematical formulation for analyzing the sensitivity of the measured response of the structure to a large number of input variables describing the influence of the environment and operating conditions [14-15]. In this context, the possibility for assessing response statistics via direct and mathematically simple post-processing of the expansion coefficients makes the PCE model a desirable tool for real-life applications [16].

PCE is a commonly employed method in uncertainty quantification, where it is often used to substitute a computationally intensive model, which is affected by random inputs, with a more cost-efficient polynomial function. In recent years it has been successfully applied to a number of civil engineering problems dealing with construction of metamodels [17-23]. In [17] PC expansion is used to represent the stochastic system output responses of three numerical modeled systems of civil bridge structures. The results obtained are compared with those from the widely used MCS and FOSM methods. The obtained PC coefficients are directly used for calculating the global sensitivity indices, which verifies the accuracy and significantly reduced computational demand of the presented method compared to the MCS-based calculation. In [18] the application of PCE for meta-modeling of dam engineering problems is explored. The response prognosis of four numerical case studies with different complexities is investigated with uncertainties propagated in material properties and modeling. The method is found as an effective technique to deal with uncertainty quantification in concrete dams. Ghanem et al. [19] focused on an embankment dam using stochastic finite element analysis involving the PCE method, where material's elastic and shear moduli are modelled as stochastic processes. The work in [20] presents a metamodeling approach designed to handle uncertainties in simulating nonlinear, dynamically evolving engineering systems. The authors utilized nonlinear autoregressive with exogenous input (NARX) models, where random parameters are used to represent uncertainty propagation within the numerical model. The random NARX parameters are expanded into a polynomial chaos. The resulting PC-NARX metamodel significantly reduces computational time while maintaining adequate accuracy. Guo et al. [21] also investigated the stability of an embankment dam using sparse PCE, assuming three soil properties—dry density, cohesion, and friction angle—as random variables. They applied both the finite difference and limit equilibrium methods to assess the dam's safety factor, presenting failure probability distributions for normal operating conditions and seismic loading. DeFalco et al. [22] introduced a method for calibrating model parameters in a Bayesian framework. This approach replaces the original model with a proxy model obtained through generalized PCE, reducing computational load while providing a global model error estimate. The method is tested on a case study of one Italian concrete gravity dam, where recorded displacements have been used to estimate model parameters values which provide a model response with minimal error. Exploring the problems of finite element model updating and structural damage identification for a small-scaled laboratory dam, the research in [23] proposed a sparse PCE method for substituting the computationally expensive FE model, enabling a low-cost and high predictive accuracy.

The application of the PCE tool for the purpose of purely data-driven diagnostics and prognosis of monitored structures was explored in [14,24]. Spiridonakos and Chatzi [24] introduced the PCE method together with the independent component analysis (ICA) tool in a long-term scale, delivering a robust performance indicator. The PCE -ICA scheme was successfully verified on damage detection for the benchmark SHM project of the Z24 bridge. Related to tower-like structures, in [14] the authors combined the PCE method with the parametric smoothness priors time varying autoregressive moving average (SP-TARMA) method. The proposed PCE-SPTARMA approach delivers a holistic model for long-term tracking of the structural behavior of two full-scale operational wind turbine structures, demonstrating the high potential of the proposed method for automated condition assessment of large real-world structures, operating in a wide range of conditions.

In recent years, significant advancements in the automation of structural monitoring systems have facilitated the collection of vast amounts of data. This has, in turn, accelerated the adoption of data-driven techniques for structural safety monitoring [25-28]. Given the limited availability of accessible monitoring data from full-scale operational engineering structures for research purposes, this study makes a key contribution by offering valuable insights into the practical implementation of a selected prognostic tool, applied to data gathered from real-world engineering structures. While previously reported studies focus on meta-modelling or multi-componential utilization of the PCE tool, herein we are testing a rather straightforward application of the PCE method, exploring direct employment of uncorrelated input set of measured environmental data and not extensively preprocessed measured response quantity, serving directly as a model output variable. The examination of the limitations and advantages of this basic methodology enhances the understanding of its limitations for practical implementation.

To this end, the applicability of the PCE method is herein tested on two distinct full-scale structures: a concrete arch dam and a steel chimney. The constructed models for both structures extrapolate on a selected single measured response variable (displacement for the dam structure and acceleration for the chimney structure), representing the model's output parameter, by incorporating the variability of measured environmental conditions (water level and ambient temperature for the dam structure and wind velocity, direction and ambient temperature for the chimney structure), serving as model inputs. The analysis of the first example of the arch dam demonstrates the effect of incomplete training set on the accuracy of the model prediction, while the second case study underlines the aspect of data condensation via the effective sensitivity analysis via the PCE-based obtained Sobol indices. The obtained results showcase the potential of the method for its efficient utilization in prognostic and diagnostic tasks within holistic and autonomous SHM frameworks.

2 Polynomial chaos expansion- theoretical background

The field of uncertainty quantification deals with systems affected by stochastic variations in system parameters, i.e. data errors which rise from limited knowledge and availability of input data, or operating (inherent) variability of the studied system. To this end, uncertainty quantification models take into account the evolution of the probability distribution of the random input. The term of "Homogeneous Chaos" was initially introduced by Wiener [29] for the purpose of modeling stochastic processes involving Gaussian random variables through the use of Hermite polynomials. To extend this approach to different types of random variables, Xiu et al.

proposed the "generalized polynomial chaos" framework [30], which adapts the method to both discrete and continuous random variables by employing orthogonal polynomials from the so-called Askey scheme [31]. In recent decades, polynomial chaos expansion has found increasing popularization in various research fields, with key developments shown in Fig. 2.1.

More specifically, if we assume the system $Y = S(X)$ is comprised of *M* random input parameters represented by independent random variables, e.g. measured water level, wind velocities and temperature values, gathered in the random vector X of prescribed joint Probability Density Function (PDF), and the output variable is of finite variance, the PCE model assumes the form [14]:

$$Y = S(X) = \sum_{\alpha \in N^M} y_\alpha \psi_\alpha(X) \quad (1)$$

where $\psi_\alpha(X)$ are polynomials dependent on multiple variables, orthonormal with respect to the probability density function f_X, $\alpha \in N^M$ is a vector of multi-indices of the multivariate polynomial basis identifying the components of the polynomials ψ_α and $y_\alpha \in R$ are the corresponding unknown deterministic coefficients of projection.

In practical applications, the sum in expression (1) is truncated to a finite sum, which is typically achieved by limiting the total maximum degree p of the polynomials in the polynomial basis to:

$$|\alpha_i| = \sum_{m=1}^{M} \alpha_{i,m} \leq p \; \forall \; i \quad (2)$$

This constraint ensures that the total number of terms in the polynomial basis will be:

$$\mathrm{P} = \frac{(M+p)!}{M!\,p!} \quad (3)$$

where *M* designates the number of random variables and *p* denotes maximum basis degree.

Polynomials dependent on multiple variables $\psi_\alpha(X)$ are obtained through the tensor product of corresponding one-dimensional orthonormal polynomials, selected based on the probability density function of the random input variables and the known Askey scheme for orthonormal polynomials [31]. Finally, the truncated PCE model to the first P terms yields a finite parameter vector y_α which may be estimated by solving Eq. (1) in a least squares sense. The least squares approach is based on minimization of the cost function *R*, estimated as sum of the squared residuals between true (observed) and modeled (predicted) system outputs.

An approach for global sensitivity analysis of PC output variables, based on computationally inexpensive post-processing of estimated PC coefficients, was proposed in the work of Sudret [16]. Exploiting the orthonormality of the PC basis and their subsequent convenient properties, the approach utilizes a variance-based sensitivity analysis tool, namely the Sobol' decomposition. Its final goal is estimation of Sobol' sensitivity indices, which represent the fraction of the total variance of the model output that can be attributed to each input variable or combinations of variables [16].

Sobol' indices are obtained as a sum of squares of the PC coefficients and represent the fraction of the total variance D of the model output that can be attributed to each input variable or combinations of variables. More precisely, the index for a single input variable X_i is called the first-order Sobol index and represents the effect of X_i alone, Eq. 4. Indices for the influence of multiple variables, such as $S_{ij}, i \neq j$, are known as higher-order Sobol indices and represent interaction effects between X_i and X_j that cannot be attributed to the individual contributions of each variable separately.

$$S_i = \sum_{\mathrm{V} \in A_i} y_\alpha^2 / D, \qquad A_i = \{\mathrm{V} \in A : \mathrm{V}_i > 0, \mathrm{V}_{j \neq i} = 0\} \quad (4)$$

3 Case study I: Concrete arch dam

The first presented case study herein is a concrete arch dam, located southwest (30 km aerial distance) of Skopje, RN Macedonia. The structure represents a thin concrete shell with double curvature, with a structural height of 64 meters. The dam is unreinforced, except for the upper third, which potentially would handle intensified oscillations in the event of an earthquake. The crest level is located at 364 meters above sea level (asl), while the lowest point at the bottom is at 300 m asl. The thickness of the crest is 2.0 m, gradually increasing to 10 m at the bottom. The structure was built over the period of six years (2006-2012), utilizing 27362 m^3 concrete for the dam body. The hydroelectric plant began operating on August 1, 2012, with an output capacity of 36.4 megawatts.

Since its construction the structure is equipped with a comprehensive monitoring system measuring: reservoir water level, underground water levels, ambient temperature and temperatures of concrete and water, rainfall, displacements at the crest and dam body, strains, rotations, accelerations during triggered earthquake events, contact stresses, etc. All aforementioned parameters are mainly

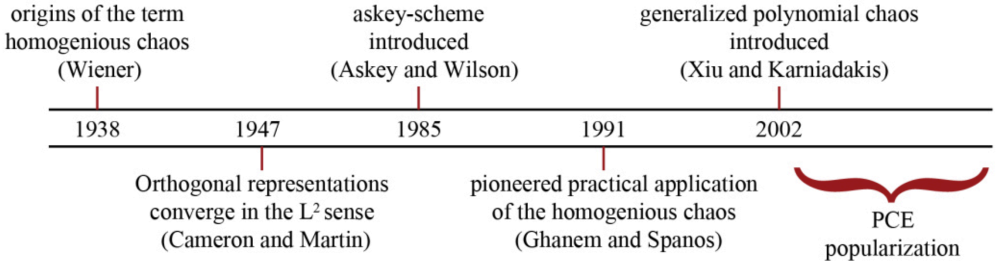

Figure 2.1. Developmental milestones for the PCE method (adopted from [4])

recorded once in six hours. The sensor distribution is based on results obtained from static and seismic analyses of a FE mathematical model, taking into account variations in the characteristics of the rock mass. This resulted in a nearly symmetrical layout of the instruments. The instruments are placed at five levels: +305.00; +320.00; +335.00, and +357.00 meters above sea level. These levels mainly correspond to points determined by the computational model at locations where the calculated values are of particular importance for the behavior of the dam. A more detailed overview of the complete acquisition system can be found in [32].

3.1 PCE application

Within this section the formulation and application of the PCE model is tested on data obtained from a two-year long monitoring period (years 2013 and 2014) for the dam structure, for a selected number of sensors, Fig.3.1.

The sampling frequency of the selected measured variables is once per six hours, which for the analyzed period of two years (excluding spurious data or missing records) provided in total of 2268 data sets, distributed as in Fig. 3.2. In order to select the appropriate physical quantities representing the influence of the environment on the structure (model input matrix), a correlation matrix has been computed as a first step. The selection of uncorrelated (or weakly correlated) input variables is a theoretical prerequisite of the PCE model [4]. Whereas various mathematical approaches do exist in transforming correlated data into uncorrelated variables [33], an advantage of the direct employment of uncorrelated (weakly correlated) input parameters is the possibility of straightforward calculation of the PCE-based Sobol' indices, demonstrated in Section 4.

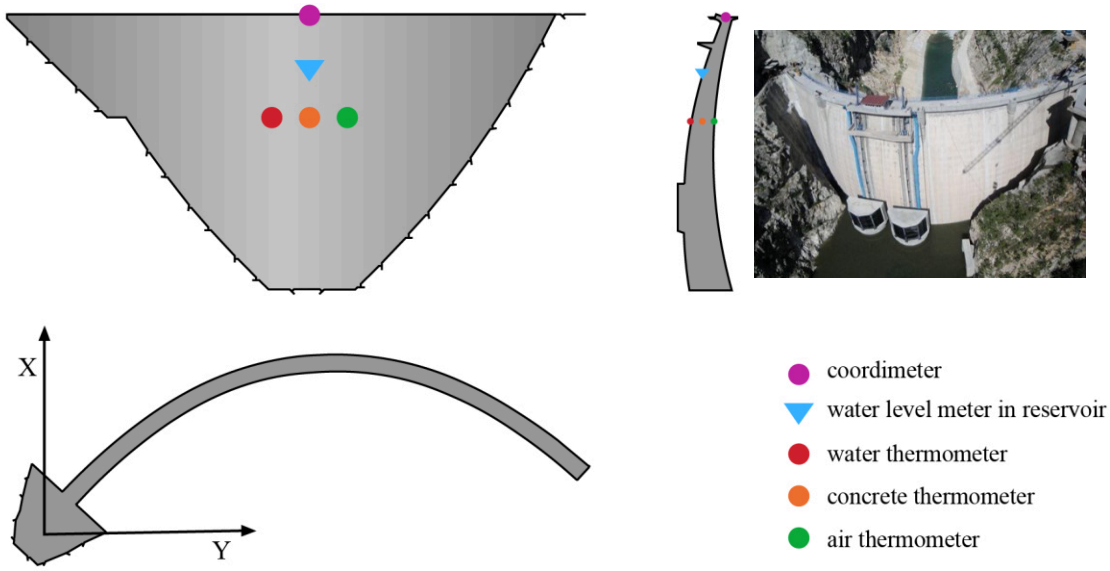

Figure 3.1. The concrete arch dam and a schematic overview of employed sensors from the installed monitoring system

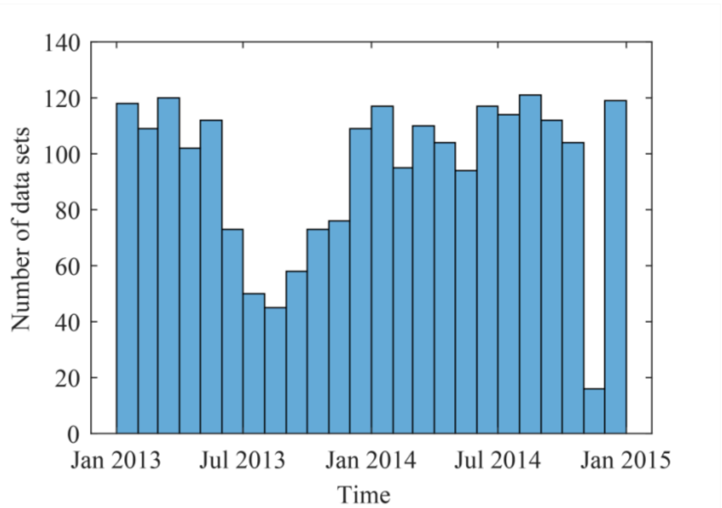

Figure 3.2. Available data sets over the two-year long period

In Fig. 3.3 the correlation plots for each pair of available input variables are presented. Due to the high correlation among the three measured temperatures, Pearson Correlation Coefficient (PCC) is 0.968 and 0.683, a single temperature-related variable, namely the ambient temperature (as most correlated to the measured displacement) was selected as an input quantity. In addition, the ambient temperature is correlated to the measured reservoir water level with the lowest PCC equal to 0.078. This enables the utilization of both quantities as PCE inputs.

On the other hand, the recorded crest displacement in X direction is selected as representing structural behavior, or as the PCE output parameter. The input/output variables are recorded at a frequency of one measurement every 6 hours, time history plots presented in Fig. 3.4.

An additional important criterion, particularly in the case of handling of a large database, is computational efficiency. In this context, the selection of the PC order, which affects the modeling precision, can also directly influence the total number of unknown PC coefficients and as a result computational time. For the assessed case study, the number of unknown PC coefficients in correlation to selected maximal PC order and the Leave One Out (LOO) error for the training and validation set of the actual case study is demonstrated in Fig. 3.5.

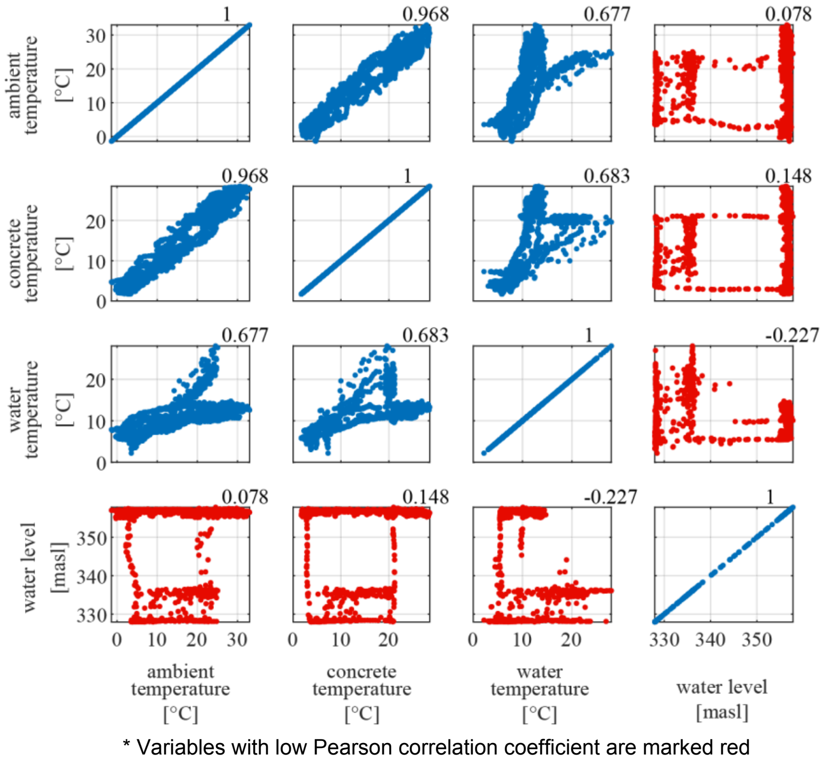

Figure 3.3. Correlation plots for the PCE input variables

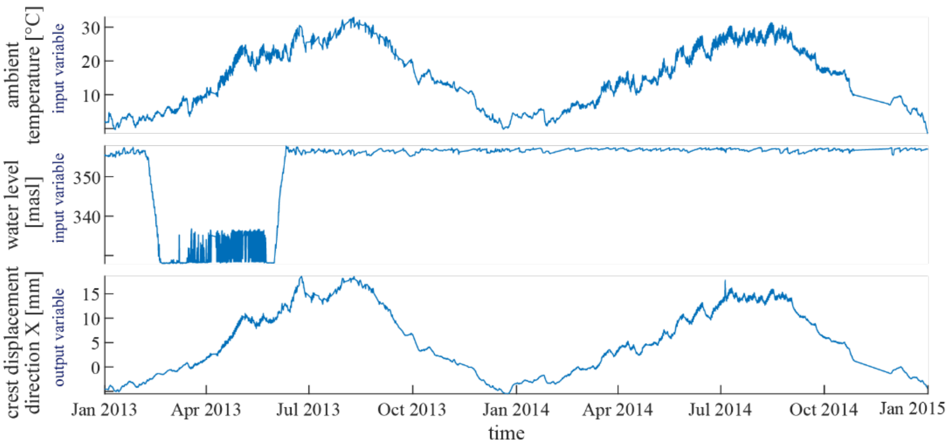

Figure 3.4. Time history plots for the selected PCE input and output quantities

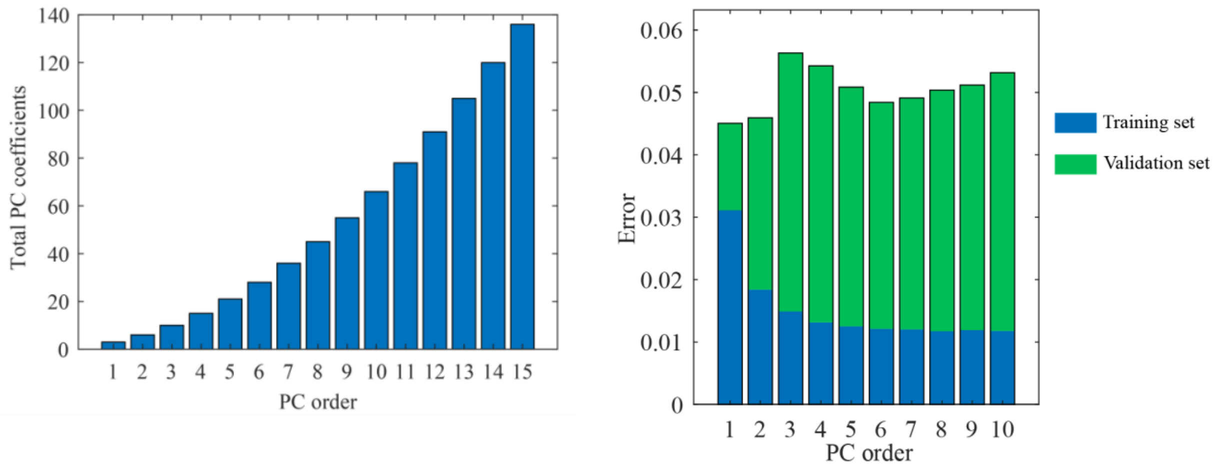

Figure 3.5. (Left) Number of unknown PC coefficients in correlation to selected maximal PC order; (Right) LOO error in correlation to PC model order for the dam case study

As a last step, the model output variable and the PDFs of the measured operational input data are fed into the PCE framework. In accordance with the PDFs of the input data, the Hermite polynomials are selected as the PC functional basis and the maximal polynomial order is selected as p=2.

In Fig. 3.6, the performance of the model with the selected output variable is demonstrated. The graphs demonstrate good alignment between the computed displacement (using the PCE model) and the actual measured displacement. The vertical dashed line marks the time interval used for training the PCE model (applied 70% of data). In the portion of the time interval (right of the vertical line) corresponding to the validation period, after training is completed, the model generates the displacement of the structure using “new” input data. From the results presented, it is evident that the model exhibits good capability in predicting the displacement of the structure in the considered direction.

The water level in the reservoir of the analyzed dam is maintained almost constant over time because, among other things, the dam is used for electricity production. To demonstrate the workings of the model to unknown data two additional analyses were performed by training the PCE model on an altered timeline, producing two different training scenarios.

In the first case (Scenario A), the training set includes data from the trial filling and emptying of the reservoir, while in the second case (Scenario B) this data is excluded. On the other hand, both scenarios include this data in the validation set. Both cases are graphically presented in Fig. 3.7, and the results of the analysis are shown in Figs. 3.8 – 3.9.

When the filling/emptying of the reservoir is not included in the training period of the model (Scenario B), the PCE validation set values drastically deviate from the monitored data. Having this in mind, in a potential autonomous SHM framework development such a deviation from "normal" expected ranges (usually $\pm 3std$) would trigger an alarm, which in this case would be a false positive (since it stems from changes in environmental conditions, not actual structural changes). This highlights the importance of using a holistic dataset during the model’s training phase, which captures the complete operational spectrum of the structure.

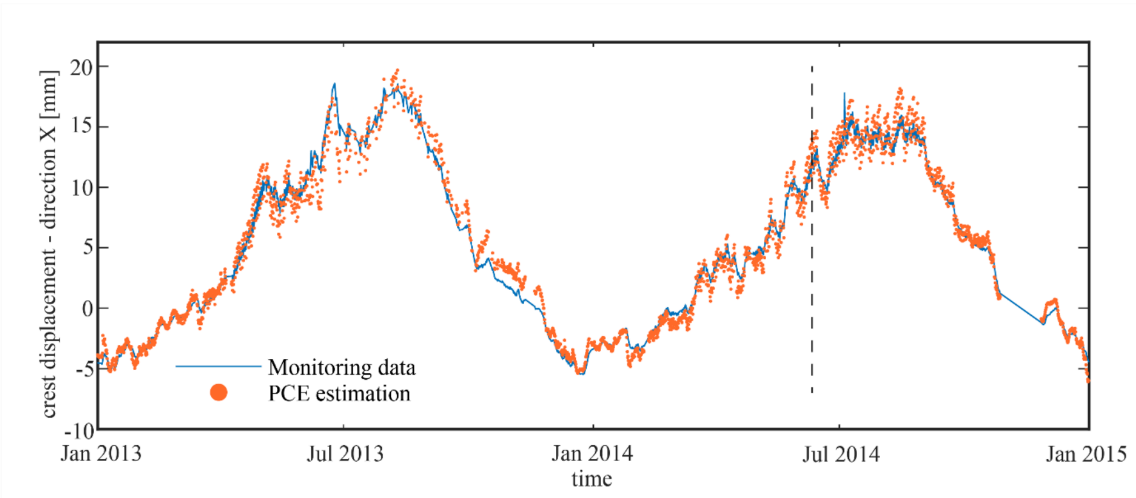

Figure 3.6. The output variable and the capability of the PCE model to predict it

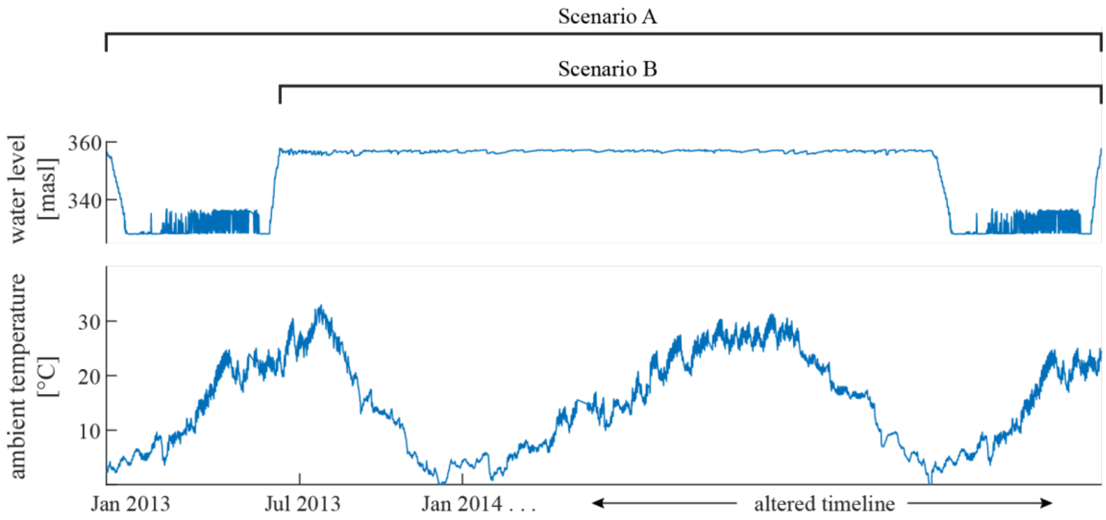

Figure 3.7. Input variables included for training for Scenario A and Scenario B

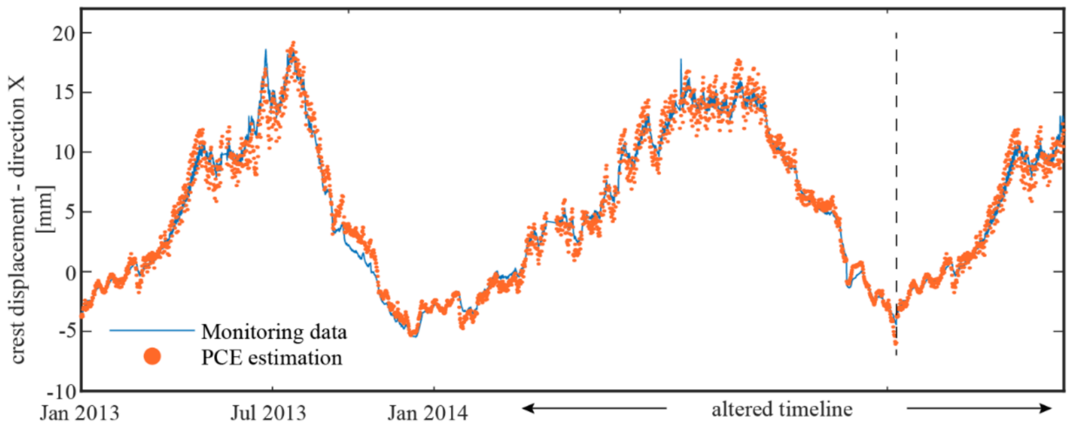

Figure 3.8. PCE model estimates for Scenario A

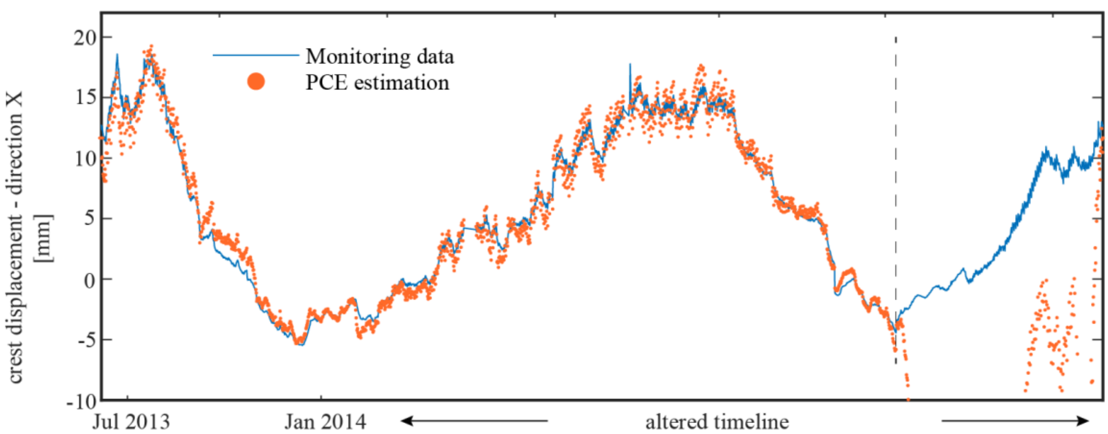

Figure 3.9. PCE model estimates for Scenario B

4 Case study II : Industrial steel chimney

The second testing case study is an industrial out of use steel chimney located at the Ohis factory in Skopje, RN Macedonia. A SHM campaign was undertaken in the period 14/12/2013 to 14/02/2014. An installed monitoring system for the complete timeframe of two months continuously measured: structural vibration responses (accelerations), environmental parameters (wind velocity and ambient temperature), as well as ground vibrations nearby the structure (Fig. 4.1).

Acceleration time histories were recorded by five tri-axial accelerometers for ambient vibration placed along the structure's height, with the sampling frequency of 200 Hz. Details on the placement and positioning of the sensors, as well as structural identification results are presented in a previous work in [34]. The authors successfully applied operational modal analysis for two identified loading scenarios: i) recurring train induced vibrations from a nearby railway, and ii) wind induced vibrations for a time frame corresponding to the maximal value of recorded wind velocity within the two months period of monitoring. The identified natural frequencies of the structure were verified with a FEM of the structure.

4.1 PCE application

In this case study, due to detected spurious trends in collected data, a three-week time frame was selected as a testbed for simplified showcasing the PC-based sensitivity analysis potential. The collected data was averaged to a sampling frequency of one record per minute, or for the analyzed period of 22 days in total 30407 data sets were used. The PCE output variable utilized to describe the behavior of the structure is the one-minute standard deviation of the measured acceleration at the top of the chimney in horizontal direction. The measured ambient temperature, wind velocity and direction were employed as PCE input variables, describing the environmental effects. The time histories and correlation plot of the selected variables are plotted in Fig. 4.2 and Fig.4.3.

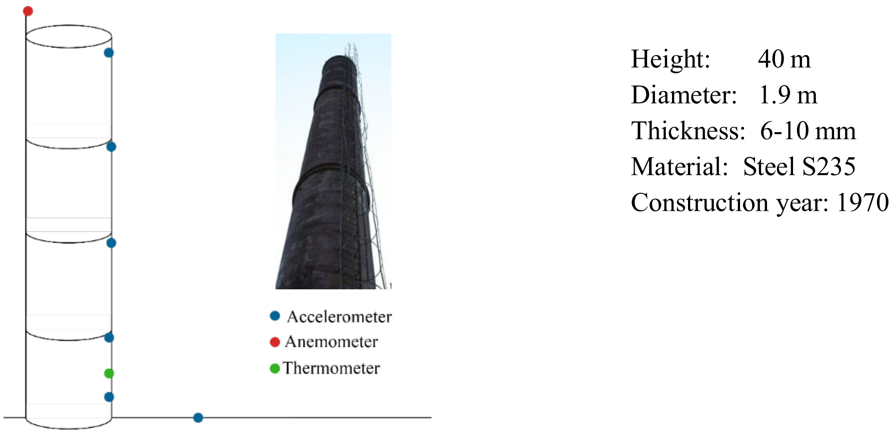

Figure 4.1. Installed monitoring system and information for the structure under study

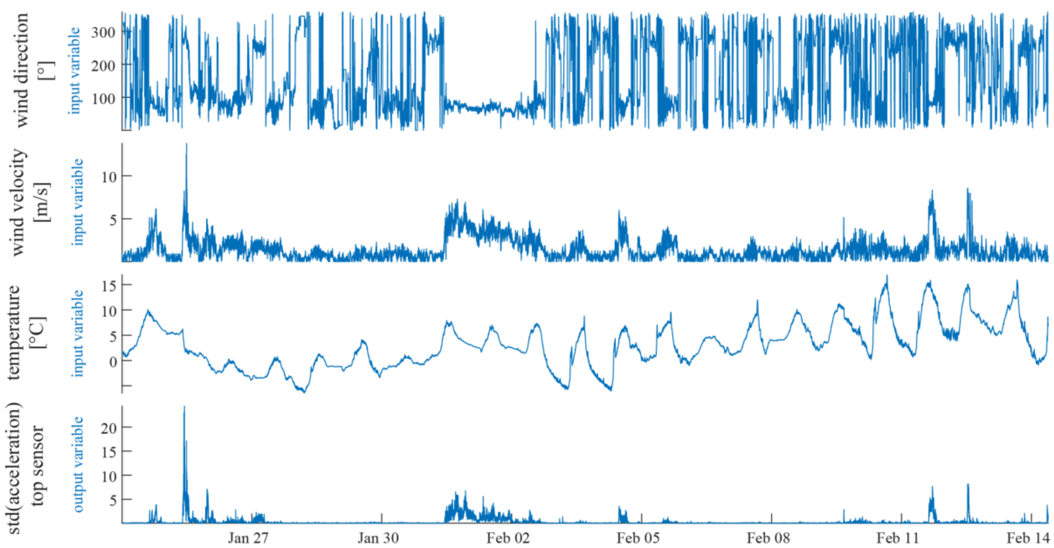

Figure 4.2. Time history plots for the selected PCE input and output quantities

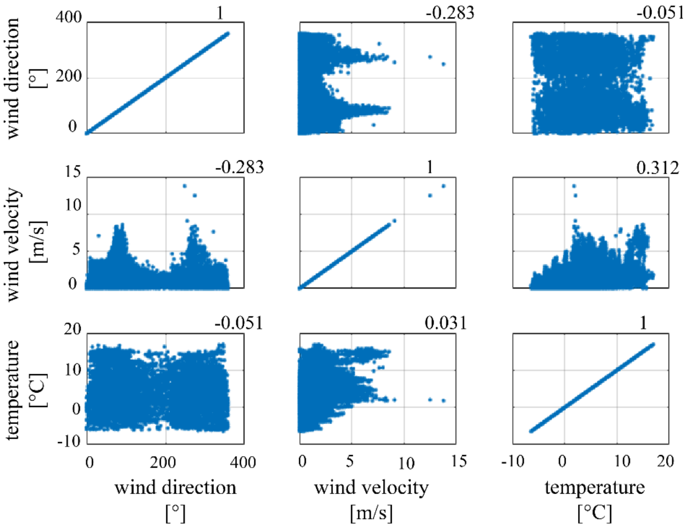

Figure 4.3. Correlation plots for the PCE input variables

In order to analyze how the variability of each individual input variable affects the chimney measured vibration, Sobol indices were calculated. The indices of first, second, and third order, along with total indices, were computed using coefficients obtained from a developed PCE data model (selected maximum PC order 4; adopted PC basis in accordance with the individual pdf of the three input variables), Fig. 4.4.

The analysis of the Sobol indices demonstrates that the variability of temperature has the least influence on the selected output representing the behavior of the structure (standard deviation of the measured acceleration) compared to the other two input variables. Therefore, a model with a reduced number of input variables is constructed, specifically with the input variables (1) wind direction angle and (2) wind velocity, Fig. 4.5. The results demonstrate that the difference between the models' LOO errors is negligibly small, specifically 6%, indicating that Model 2 (the model with a reduced number of input variables) has a slightly lower average error. This confirms the advantage of the Sobol indices analysis, as they potentially can reduce the dimensionality of the problem and enable a balance between computational efficiency and modeling precision.

In addition to Fig. 4.5., a look at the comparison of the PCE estimated values for the validation set for the both models in Fig. 4.6 (b) shows that model 2 in general produced higher values of the amplitudes of the modeled output parameter in comparison with model 1, which serves well for the estimated higher peaks of the std of the acceleration. Both models, however, perform similar for the training sets Fig. 4.6 (a).

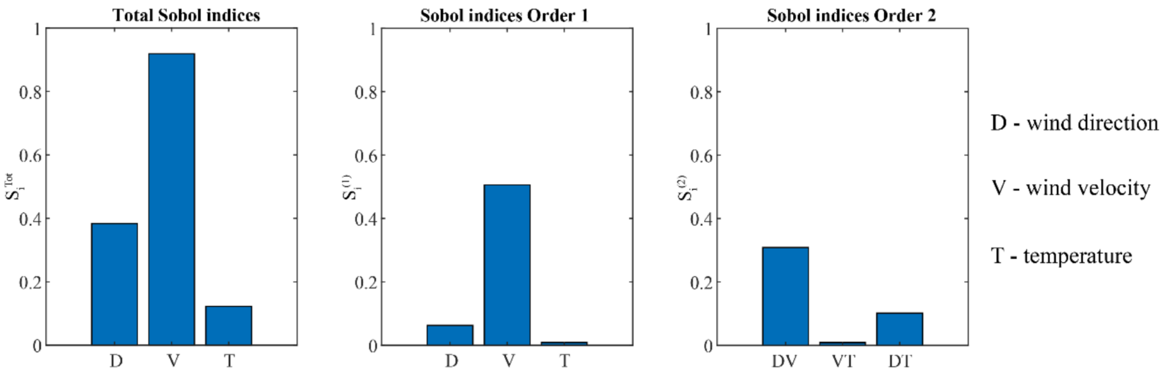

Figure 4.4. Calculated PCE-based Sobol indices

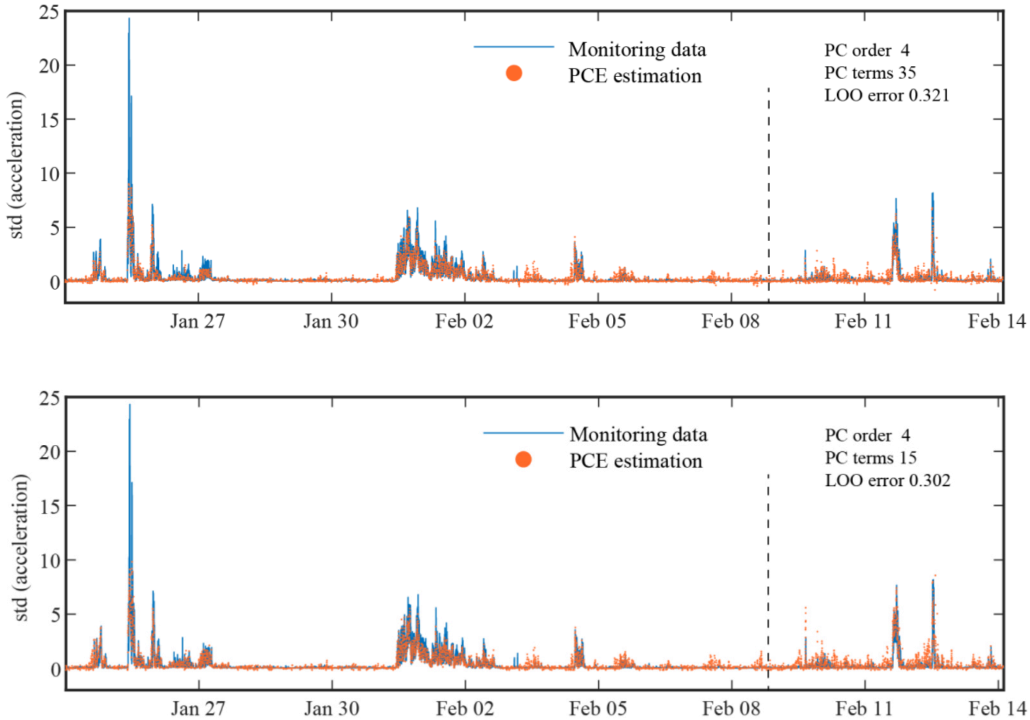

Figure 4.5. (Top) PCE model estimates for three input variables;
(Bottom) PCE model estimates for two input variables;

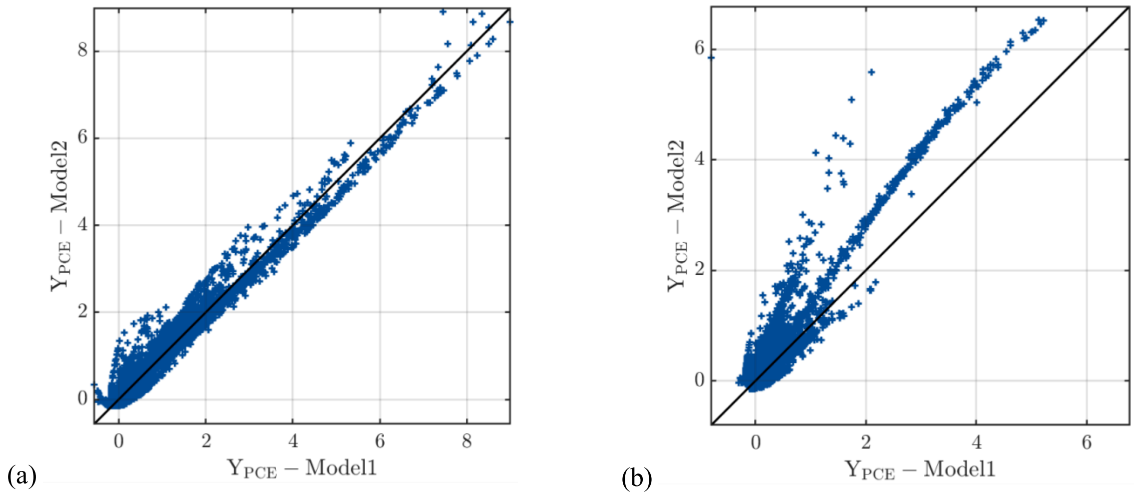

Figure 4.6. (a) PCE model 1 estimates versus PCE model 2 estimates - training data;
(b) PCE model 1 estimates versus PCE model 2 estimates - validation data;

5 Conclusions

The research study presented herein focused on the construction and implementation of two separate data models using the polynomial chaos expansion method. Both models were successfully applied on two different full-scale case studies, namely a concrete arch dam and an industrial steel chimney, demonstrating the significant potential of the method to be used as a tool for long-term monitoring of engineering structures. In effort to present the workings of the method a rather straightforward utilization of the models was tested, i.e. featuring uncorrelated input sets and crude application of measured output parameters.

The obtained results demonstrated that the method represents an efficient tool for constructing data-driven models, characterized with simplicity of construction and little parameter tuning required. Specifically, after selecting the appropriate type and maximum degree of the polynomials in the basis, the coefficients are obtained through simple matrix algebra. Additionally, the obtained results with PC-based estimated Sobol's indices have accentuated the convenience of the tool for sensitivity analysis and as a result reducing the dimensionality of the studied problems.

Future research efforts are extended towards incorporating mutually dependent input variables aided by additional mathematical tools which will ensure preservation of the physical meaning of the transformed variables, as well as practicing the PCE tool with an accompanying diagnostic SHM tool which will allow for improved tracking of changes in structural responses.

CRediT authorship contribution statement

Viktor Georgijev: Writing- original draft, Formal Analysis, Resources, Visualization.

Simona Bogoevska: Writing-review & editing, Conceptualization, Methodology, Resources, Visualization, Validation, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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